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Buoyancy driven flow from a waning source through a porous leaky aquifer

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A R T I C L E   I N F O

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A B S T R A C T

We develop a series of models to describe the migration of a buoyant fluid through a layered permeable rock following release from a localized waning source. In particular, if the fluid is injected into a high permeability layer, bounded above by a layer of lower permeability, a plume migrates along the interface, with some draining into the low permeability layer if the current is sufficiently deep to overcome the capillary entry pressure. We show the motion of the fluid is controlled by a number of key factors with the dominant dimensionless numbers being the time-scale for the source to decay compared to the time-scale for draining through the low permeability layer, the residual saturation of the gas and water in the formation as that phase is displaced by the other phase, and the capillary entry pressure, as measured by the critical depth of the current required for draining, as compared to the initial depth of the current. Simplified analytical models are presented to illustrate some of the key controls on and transitions in the flow, and the models are used to explore leakage and trapping prior to flow reaching a fault zone.

1. Introduction

There is growing interest in the migration of gas from a localized source through a permeable rock owing to its relevance for the dispersal of CO_2 sequestered in the subsurface, but more generally for the dispersal of buoyant fluids which may form in and migrate from geological waste repositories (Bickle et al., 2007; Hesse et al., 2007). In this latter situation, an interesting feature is that the rate of generation of buoyant fluid may progressively decay over a period of several hundred years. The gradual waning of the source leads to some interesting dynamical balances in the migrating buoyant plume, and has a critical impact on the dispersal pattern of the fluid; this forms the topic of the present contribution.

Several models have been developed to describe the motion of buoyant plumes of fluid migrating through the subsurface (Barenblatt, 1996; Bear, 1972; Huppert and Woods, 1995), and recently these have been extended to include the equations for two-phase flow in a permeable rock, including the effects of the relative permeability between the two phases (Hesse et al., 2006; Nordbotten and Celia, 2006). With a localized source of buoyant fluid, these models lead to the prediction that in a confined aquifer, a buoyant plume develops adjacent to the upper boundary of the aquifer and then spreads out along the aquifer (Hesse et al., 2006, 2008; Mitchell and Woods, 2006). If it comes into contact with a fault/fracture system, then some of the flow may drain upwards along the fault where it may then intersect another permeable layer, enabling part of the leakage flux to continue spreading laterally (Pritchard et al., 2001; Pritchard, 2007). If the source wanes, the continuing finite plume will then develop a trailing front. As this advances through the formation, there may be some capillary trapping of the fluid leading to a residual saturation (Barenblatt, 1996; Hesse et al., 2006; Obi and Blunt, 2006; Kharaka et al., 2006; Farcas and Woods, 2009a). As a result, the plume becomes progressively depleted as it migrates through the formation, leaving the capillary trapped zone behind.

In the present contribution, we examine the motion of a buoyant plume supplied by a waning source which spreads through a permeable rock, bounded above by a less permeable thin layer into which the fluid may slowly drain off. We also account for the capillary retention of a fraction of the fluid at any point along the plume where the flow thickness decreases in time. This leads to predictions of the fraction of the current which may remain trapped in the original layer rather than leaking off higher into the formation.

We note that our analysis is restricted to a two-dimensional flow, in order to identify some of the key controls on the system, although we note that three-dimensional cross-flow effects can also arise, especially far upslope of the source (cf. Vella and Huppert, 2007; Farcas and Woods, in press). However, with a long linear source, the effects of three-dimensional spreading of the flow

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upslope of the source to points beyond the extremities of the source, may only become dominant once the flow has advanced a substantial distance upslope, and so in that case, the present modelling may provide a reasonable approximation to the flow in the near field. Also, in some situations, the flow may be structurally confined such that the two-dimensional model may provide a reasonable leading order model for the flow.

2. The model

We consider the migration of a buoyant fluid of density r through a permeable layer of rock, saturated with fluid of density r + Dr, which is bounded above by a thin layer of lower permeability. We assume the injected fluid is only able to invade the low permeability layer if it is sufficiently deep, $h(x,t)$, to overcome the capillary entry pressure, $h > h_c$. Otherwise it will continue to run upslope through the high permeability layer, under the lower boundary of this 'seal' layer (Fig. 1). As the current spreads out along this layer, of inclination to the horizontal *q*, the alongslope motion is governed by the buoyancy forces acting on the flow, according to the relation for the transport or Darcy flux μ (cf Bear, 1972; Barenblatt, 1996)

$$u = -\frac{k\Delta\rho g}{\mu}\left[\frac{\partial h}{\partial x}\cos\theta - \sin\theta\right] \tag{1}$$

where $h(x,t)$ is the thickness of the current, x is the alongslope position, *m* is the viscosity and k is the effective permeability of the fluid as it migrates through the rock, where we account for the effects of relative permeability in a very simple fashion with the single permeability parameter. Most of the interest in this work is in modelling gas or supercritical fluid dispersion and we model the motion through the rock in terms of an effective permeability. This has been shown to give good leading order predictions compared to the full two-phase flow relations for such buoyancy driven flows (cf. Nordbotten and Celia, 2006; Hesse et al., 2006). The rate of loss of fluid from the current to the overlying low permeability layer, through unit length of the boundary, depends on the permeability k_b, the thickness b of the seal layer, and the thickness of the current (cf. Pritchard et al., 2001) according to the relation

$$\text{Loss} = \frac{k_b g\Delta\rho(h+b)\cos\theta}{b\mu} \tag{2}$$

These equations are then combined with the relation for the conservation of mass, which in the invading flow has the form

$$\phi(1 - s_W(1-R))\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}[hu] - \text{Loss} \;\; \text{for} \;\; \frac{\partial h}{\partial t} > 0 \tag{3}$$

since as the invading gas advances into the formation, there is a fraction s_W of the pore space which remains saturated in the original fluid, and gas then dissolves into this fluid, representing an effective additional pore volume $fs_W R$ for the injected fluid. Here R denotes the mass fraction of gas dissolved in the original fluid (which occupies the fraction s_W of the pore volume) multiplied by the density of the original fluid and divided by the density of the free gas phase. Also, in this expression *f* denotes the porosity of the rock.

In contrast, in any part of the flow where the depth of the current decreases with time, then as the buoyant fluid vacates the pore space and is displaced with water, there will be some residual gas trapped which occupies a fraction s_g of the pore spaces. Here, for simplicity, we model this as being a constant (cf. Barenblatt, 1996; Hesse et al., 2006, 2008) and so the conservation of mass takes the form

$$\phi(1 - s_W - s_g)\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}[hu] - \text{Loss} \;\; \text{for} \;\; \frac{\partial h}{\partial t} < 0 \tag{4}$$

In order to solve for the motion of the current we require some boundary conditions. First, it follows that the nose of the current propagates at the rate

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{u}{\phi(1 - s_W + Rs_W)} \tag{5}$$

while we assume that the source flux, at $x = 0$, gradually wanes, at a rate

$$Q(t) = Q_o\exp(-t/\tau) = \frac{k\Delta\rho g\sin\theta}{\mu}\left[1 - \cot\theta\frac{\partial h(0,t)}{\partial x}\right]h(0,t) \tag{6}$$

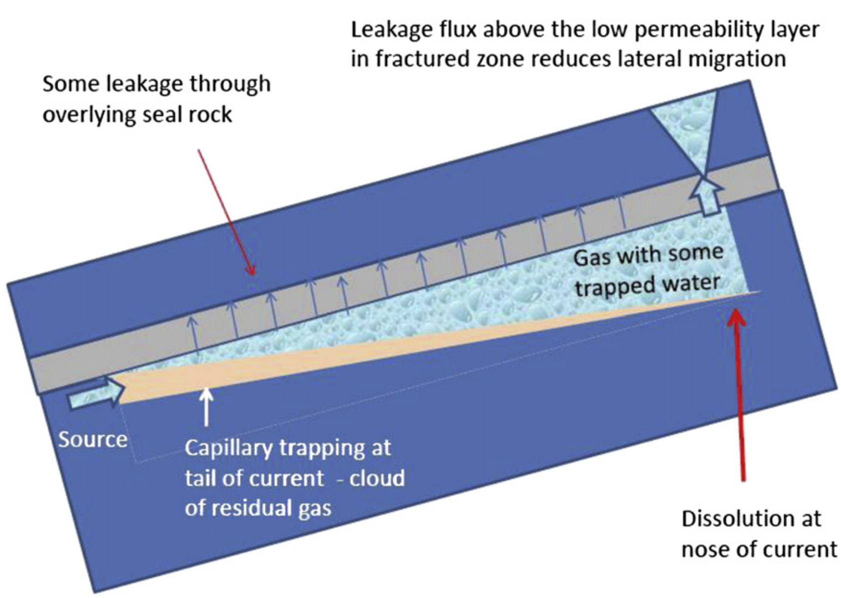

Fig. 1. Cartoon of the flow geometry for the present problem.

where t is the e-folding time over which the source flux decays. It is this waning source flux, coupled with the dynamics of continuous leakage of fluid through the overlying seal rock, and the capillary retention at the tail of the current, which provides the new analytic results of this paper.

From eq. (6), we deduce that there is no drainage if the source flux Q_o is smaller than a critical value

$$Q_o < \frac{h_c g \Delta\rho \sin\theta}{\mu} = Q(\text{crit}) \tag{7}$$

3. Approximations and analytical solutions

With this system of equations, we can now develop a series of solutions for the motion of the plume of gas along the inclined low permeability layer. These solutions are useful for exposing some of the key controls on the distance travelled along the layer, and also how the current partitions between that component which is retained in the original layer and that component which migrates through the low permeability partial seal layer and higher into the formation.

Before developing solutions for the motion, there are some simplifications which we can introduce which simplify the analysis, and allow for an approximate analytical solution. First, as the current spreads out and disperses into a relatively long and thin flow, of lateral scale L and depth H say, such that $L > H \cot q$, then the alongslope component of gravity, proportional to $\sin q$, dominates the force associated with the cross-slope component of gravity which acts on variations in the alongslope depth of the current and is proportional to $\cos q\, \partial h/\partial x$. In this limit, $h \cot q < \text{L}$, the dynamical term proportional to $\cos\theta\partial/\partial x(h\partial h/\partial x)$,which arises in the first term on the right had side of eqs. (3) and (4), as may be inferred by combining these eqns with eq. (1), can be neglected. Indeed, we demonstrate that our analytical solutions are consistent with this approximation.

Also, in the limit that the critical current depth required for draining, h_c, satisfies $h_c > b$, the depth of overlying low permeability layer, then the numerator $(h + b)$ in the loss term can be approximated by h ($>h_c$) since the loss only arises if the flow is sufficiently deep to overcome the capillary entry pressure (cf. Woods and Farcas, 2009a).

It is also convenient to introduce the scaling for the speed

$$U = \frac{k\Delta\rho g \sin\theta}{\mu\phi(1 - s_w - s_g)}, \tag{8}$$

the dimensionless ratio of the speed of the front and the tail of the current, as given by

$$\lambda = \frac{1 - s_w(1 - R)}{1 - s_w - s_g} \tag{9}$$

and the inverse of the time-scale for the draining flux

$$\beta = U\frac{k_b \cos\theta}{kb\sin\theta} \tag{10}$$

With a waning source, these approximations lead to the governing equations

$$\lambda\frac{\partial h}{\partial t} = -U\frac{\partial h}{\partial x} - \beta h \text{ for } h > h_c \text{ and } \frac{\partial h}{\partial t} > 0 \tag{11}$$

$$\lambda\frac{\partial h}{\partial t} = -U\frac{\partial h}{\partial x} \text{ for } h < h_c \text{ and } \frac{\partial h}{\partial t} > 0 \tag{12}$$

and

$$\frac{\partial h}{\partial t} = -U\frac{\partial h}{\partial x} - \beta h \text{ for } h > h_c \text{ and } \frac{\partial h}{\partial t} < 0 \tag{13}$$

$$\frac{\partial h}{\partial t} = -U\frac{\partial h}{\partial x} \text{ for } h < h_c \text{ and } \frac{\partial h}{\partial t} < 0 \tag{14}$$

with the boundary conditions that at the source,

$$(1 - s_w - s_g)Uh(0,t) = Q_o \exp(-t/\tau) \tag{15}$$

and that at the nose of the current, $x = x_n(t)$,

$$h(x_n,t) = 0 \text{ and } \frac{dx_n}{dt} = \frac{U}{\lambda} \tag{16}$$

It may be seen from the definition of λ that $\lambda > 1$, and so the advection speed of the current in the region in which the current is invading new rock, U/λ, is slower than the advection speed of the current in the region in which it is receding from the rock, U. This means that the nose of the current in which the depth decreases from a maximum to zero, occurs across a localized region whose detail depends on the cross-slope component of gravity. In this simplified model, this is represented by a localised front at $x = x_n(t)$. The structure of the current behind this front depends on the source flow rate compared to the draining rate, and we now consider a range of cases in turn.

3.1. Small supply flux with no leakage current

If $Q < Q(\text{crit})$, then there is no drainage into the overlying layer, and as the current moves forward along the boundary, the source flux gradually wanes, leading to a waning plume. In this case, with the draining term neglected, the solution of the equation for the current depth, eq. (14), can be written in the form

$$h(x,t) = h(0,0)\exp\left[-\frac{t}{\tau} + \frac{x}{U\tau}\right] \tag{17}$$

It follows that surfaces of constant depth advance forward at a speed given by U (*eg* see Fig. 2 below). This is faster than the speed of the leading front, U/λ, and so the leading edge of the current gradually becomes shallower with time. This is a result of the loss of fluid through capillary retention as the depth of the current at a given point in space behind the leading front gradually decreases in time.

Indeed, by direct substitution, it follows that the depth of the current at the leading edge, $x_n = Ut/\lambda$, has the form

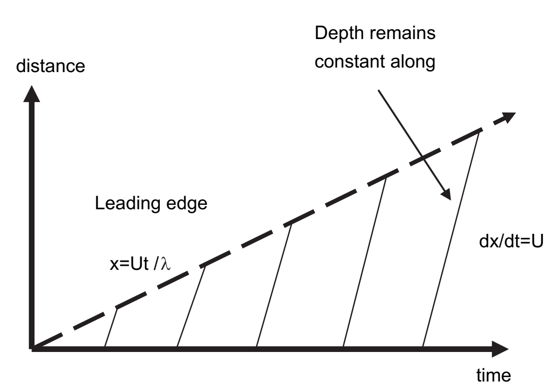

Fig. 2. illustrates the evolution of the flow in terms of the motion of surfaces of constant depth in the x–t plane.

$$h(Ut/\lambda, t) = h(0,0)\exp\left[-\frac{(\lambda-1)t}{\lambda\tau}\right] \quad (18)$$

We can also calculate the area of the zone of the rock which is invaded by the current. This can be used to estimate the volume of the residual fluid which is trapped in the pore space once the current recedes, although we note that, in time, this trapped fluid may dissolve into the water, and be carried off by any hydrological flow. The solution above for the shape of the current as a function of distance and time illustrates that at each point in space, the current is deepest on first arriving at that point. The current first arrives at each point x after a time lx/U (Fig. 2), and so the maximum depth of the current at a distance x from the source, $h_{max}(x)$, is

$$h_{max}(x) = h(0,0)\exp\left[-\frac{x(\lambda-1)}{U\tau}\right] \quad (19)$$

This curve describes the locus of the zone in which there may be some residual plume fluid once the plume has drained and moved on, and hence in which there may be a possible source of contaminant in a subsequent hydrological flow. In Fig. 3 below, we illustrate the envelope of the zone contaminated with gas and compare this with the instantaneous profiles of the buoyant plume at different times.

3.2. Larger source flux and drainage

With a larger source flux, $Q > Q(\text{crit})$, then initially there will be some drainage into the overlying layer, with $h > h_c$. In the region of the current where $h > h_c$ the solution may be written in the form,

$$h(x,t) = h(0,0)\exp\left[-\frac{t}{\tau} + \frac{x}{U\tau}(1-\beta\tau)\right] \quad (20)$$

and the rate of propagation of surfaces of constant depth is now faster than in the case with no draining, as the fluid leaks off through the overlying layer. We will now see that these new solutions are very different from the case with no draining (sect 3.1)

3.2.1. Slow draining or rapid decay of the source flux

In the case $1 > \beta\tau$, the depth of the current h increases with distance from the source at a given time since the draining of the fluid is slow compared to the decay of the source and hence the fluid at the source has the smallest flux and so is shallowest; as a consequence, the depth first decreases to value $h = h_c$ at $x = 0$ when $t = t_c$, as given by (see Fig. 4 and 5)

$$t_c = -\tau \ln\left(\frac{h_c}{h(0,0)}\right) \quad (21)$$

Subsequently, as the flux continues to wane, the depth of the current near the source decreases to values $h < h_c$ (Fig. 5) and the location of the point at which the depth has value h_c migrates away from the source and has position $x = x_c(t)$. In the region $0 < x < x_c$, the current does not drain since $h < h_c$. As the zone in which $h < h_c$ advances outwards from the source, the leading part of this region, where $h = h_c$ has position given by (Fig. 4)

$$x_c(t) = \frac{U(t-t_c)}{(1-\beta\tau)} \quad (22)$$

Meanwhile the leading edge of the current has position $x_n = Ut/\lambda$ (cf. eq. (16)), and so the zone in which draining occurs advances progressively further from the source. Eventually, the front $x = x_c(t)$,which represents the closest point to the source at which the depth has value h_c and hence can drain, reaches the leading edge of the flow. This occurs at time

$$t = t_d = t_c\left[1 + \frac{\beta\tau}{\lambda} - \frac{1}{\lambda}\right]^{-1} \quad (23)$$

Subsequently, there is no more draining anywhere in the flow (Fig. 4). For times $t > t_c$ the closest point to the source at which the depth has value h_c is given by the front $x = x_b(t)$ where

$$X_b(t) = U(t-t_c) \quad (24)$$

This lags behind the front $x = x_c$, and between these fronts, in the region $x_b < x < x_c$, the depth has the constant value h_c but there is no draining. In the near source region, $0 < x < x_b$, in which the current depth $h < h_c$, the plume has shape (cf. eq. (17) and Fig. 4)

$$h(x,t) = h(0,0)\exp\left[-\frac{t}{\tau} + \frac{x}{U\tau}\right] \quad (25)$$

In this near source region, the depth increases with distance from the source, and reaches the critical depth $h = h_c$ at the point $x_b(t)$. Eventually, at time t_b, the nearest point to the source at which the current increases to depth h_c, as given by $x = x_b$, reaches the leading edge of the current, so that $x_b(t) = x_n(t)$. This occurs at time $t = t_b$ given by

$$t_b = \lambda t_c/(\lambda - 1) \quad (26)$$

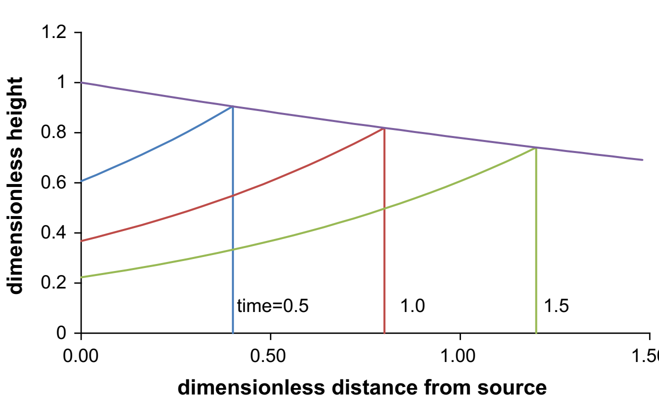

Fig. 3. Comparison of the envelope of the zone invaded by the injected fluid and the instantaneous shape of the injected fluid plume at times 0.5τ, 1.0τ and 1.5τ.

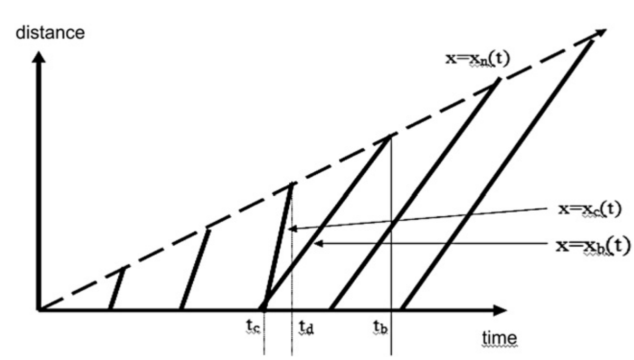

Fig. 4. (x,t) plot to illustrate the evolution of the fronts $x = x_b$, x_c and x_n as they evolve with time in the current. At times earlier than t_c the current is deeper than h_c, but for times greater than t_b the whole current is thinner than the critical depth h_c. For intermediate times, the near source region, $x < x_b$, is shallower than h_c while the more distal parts of the current are either of depth h_c, for $x_b < x < x_c$ or of depth greater than h_c for $x_n > x > x_c$.

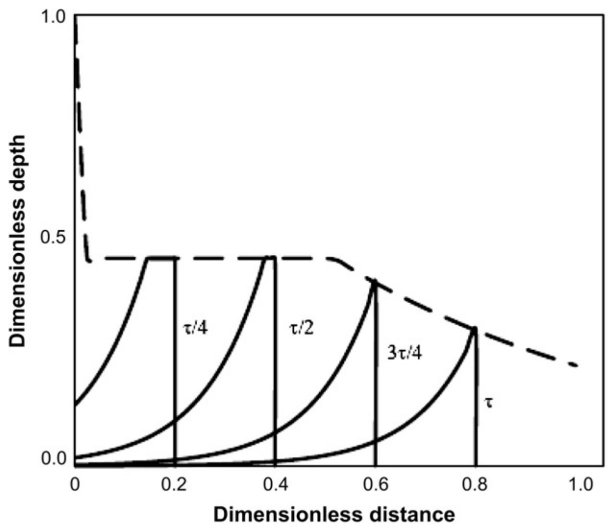

Fig. 5. Illustration of the evolution of the current with time. Each profile corresponds to a vertical line (ie constant time) in the (x,t) plane of Fig. 4; in this case, the profiles at $t/4$ and $t/2$ lie in the range $(t_d < t < t_b)$, while the profiles at $3t/4$ and t correspond to times greater than t_b.

Subsequently the current is described by relation (17), and is everywhere shallower than h_c. This sequence of flow regimes is illustrated in the Fig. 4 and 5 shown below.

3.2.2. Fast draining or slow decay of the source

In the case that $1 < \beta\tau$, and with $Q > Q_c$, the current is initially deeper than the critical value for draining, $h > h_c$, and the flow initially advances with profile

$$h(x,t) = h(0,0)\exp\left[-\frac{t}{\tau}+\frac{x}{u\tau}(1-\beta\tau)\right] \tag{27}$$

in the region $0 < x < Ut/\lambda$ (Fig. 6). In this case, at a given time, the current becomes shallower with distance from the source, as a result of the draining occurring more rapidly than the rate of decay of the source, so that the flow further from the source has less flux than that at the source (Fig. 7, dashed lines). As a result, the leading front of the current eventually reaches the critical depth at which draining ceases, $h = h_c$. This occurs when

$$t = t_d = t_c\left[1+\frac{\beta\tau}{\lambda}-\frac{1}{\lambda}\right]^{-1} \tag{28}$$

Subsequently, the leading edge of the current continues forward with depth $h = h_c$ while the closest point to the source at which the depth of the current $h = h_c$, as given by $x = x_c$, migrates backwards towards the source according to the relationship (cf. eq. (22) and Fig. 6)

$$x_c = U\frac{t-t_c}{1-\beta\tau} \tag{29}$$

This front eventually reaches the source when $t = t_c$. (Fig. 6). Subsequently, for $t > t_c$, the depth of the current at the source decreases to values $h < h_c$ and is given by the original solution (17) in the region $0 < x < x_b(t)$. From this solution, it follows that the point nearest to the source at which the current depth equals the critical depth h_c has position

$$x_b = U(t-t_c) \tag{30}$$

This front eventually catches up with the leading edge of the current, which advances at the rate
$X_n = Ut/\lambda$, at the time given by (cf. eq. (27) and Fig. 6)

$$t = t_b = \lambda\frac{t_c}{\lambda-1} \tag{31}$$

Subsequently, the whole flow evolves according to the simple non-draining solution (17) (see Fig. 7, dotted line).

3.3. Fraction which drains

In general the fraction of the flow which drains depends on the capillary pressure, which suppresses the draining, the source flow rate, the ratio of the draining time to the decay time of the source, $\beta\tau$, and also the residual saturation of the water and the gas at the advancing and receding fronts, as expressed by λ.

In general the expression is complex to calculate, but is found by comparing the volume input at the source with the volume which remains in the formation, with the difference representing the fraction which has drained. There is however a useful limit when $h_c < h(0)$ in which case, to leading order, the fraction retained in the original layer may be found by integration of eq. (20) evaluated at $t = lx/u$. This leads to the result that the fraction of the source fluid which remains trapped in the formation, F, is given by

$$F = \frac{\lambda-1}{\lambda-1+\beta\tau} \tag{32}$$

where we note that $\lambda > 1$ (eq. (9)). This expression effectively compares the process of capillary trapping at the nose and tail of the flow with the drainage through the upper boundary. It illustrates that if the draining time $1/b$ is short compared to the decay time of the source, t, then F will become relatively small, and much of the injected fluid can drain away, whereas if the draining time is comparable to or longer than the decay time of the source, then much of the injected fluid remains in the original layer. As the capillary pressure increases, this further restricts the fraction of the flow which drains, and so the above expression provides a lower bound on the fraction of the flow which remains trapped in the original layer of the formation.

We illustrate the variation of F with $\beta\tau$ for a series of representative values of λ (0.05, 0.1 and 0.15) in Fig. 8 below.

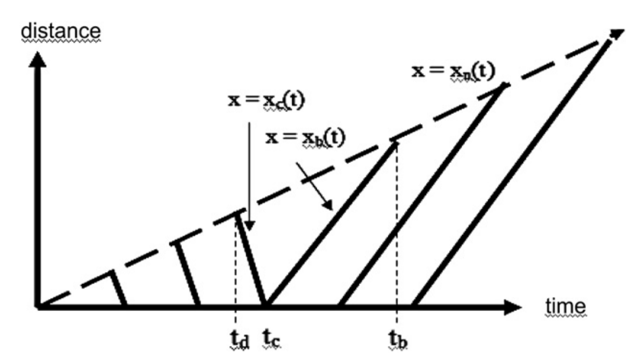

Fig. 6. Illustration of the structure of the current on an x–t plot. The figure shows how the various transition points in the current evolve with time. For $t < t_d$ the flow is everywhere deeper than h_c. For $t_d < t < t_c$ the near source region is deeper than h_c while the distal part of the flow, $x > x_c$, has constant depth h_c. For $t > t_c$ the near source region has become shallower than h_c, while for $x > x_b$ the flow has constant depth. At late times, $t > t_b$ the flow is everywhere shallower than h_c.

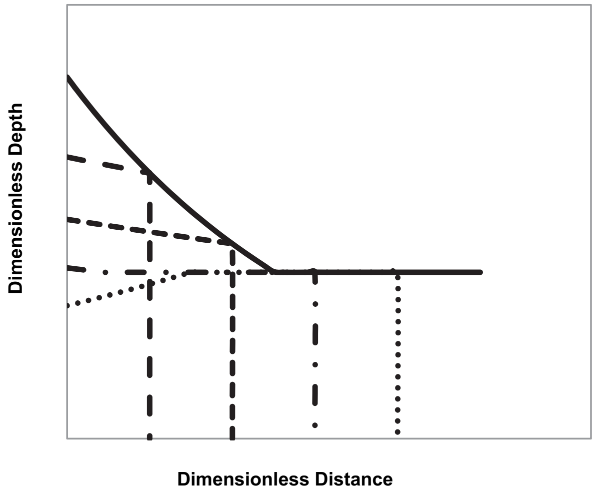

Fig. 7. Illustration of the case in which for early times, $t < t_d$, there is a draining zone near the source (long and short dashed lines) and that as the current loses mass through draining it reaches the critical depth at the nose of the flow, $x = x_n$, when $t = t_c$. For $t_c > t > t_d$ the location of the point closest to the source at which the depth equals the critical value h_c progressively migrates back to the origin which it reaches at $t = t_c$ (dot-dashed line) . For $t > t_c$, the depth is smaller than h_c in the region $x < x_b$, while the more distal part of the flow, $x > x_b$, has constant depth h_c (dotted line). Eventually, for $t > t_b$, the whole flow is shallower than h_c.

4. Draining through faults

In comparison with the above results in which the draining occurs through the upper boundary of the formation, we now consider the case in which fluid leaks off through a localised fault which cuts across the layers. Typically faults are narrow compared to the length scale of the flow, but provide a higher permeability route to the surface. If the fault connects the flow in the lower flowing layer to a layer of high permeability above the 'seal' layer, then the flux through this fault will have the form (cf. Pritchard, 2007)

$$Q_{\text{fault}} = \frac{k_f \Delta\rho g \cos\theta\, hw}{\mu b} = \Omega h \tag{33}$$

where w is the width of the fault, and b the vertical thickness of the fault, across which the gas pressure acts to drive the flow through the fault. k_f is the permeability of the fault, and h is the current thickness just upstream of the fault. Here we assume that $h > b$ so that the hydrostatic pressure driving the flow through the fault is

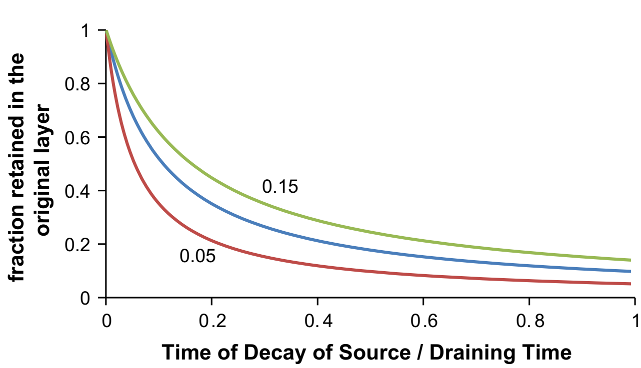

Fig. 8. Illustration of the variation of the fraction of the flow which remains trapped in the original layer of the formation as a function of the time of decay of the source compared to the draining time across the thin partial seal layer.

associated with the buoyancy of the plume of injected fluid in the lower flowing layer.

The fault flux Q_{fault} represents a discontinuity in the flux of gas along the layer. Since the alongslope flux scales as Uh, it follows that the drainage through the fault dominates the flux along the formation if $\Omega > U$, and in this case, there will be no flux beyond the fault, which for convenience we assume is located at $x = x_f$.

In this case, the fraction which remains in the formation is given by the fraction which is trapped by capillary retention upstream of the fault

$$F = \left(1 - \exp\left[-\frac{x_f(\lambda - 1)}{U\tau}\right]\right) \tag{34}$$

Here, the critical balance is between the distance the fault lies away from the source and the distance that would be travelled by the plume over the time required for the source to decay, $U\tau$.

5. Application

It is useful to examine the implications of the model for a typical example of the flow in a layered permeable rock. In the case of a geological waste repository, there may be a flux of buoyant gas with a decay time of order 300 years, and an initial flux of 10^{-5} m^2/s per unit length of the repository. If there is a layer of rock of permeability 10^{-15} m^2 bounded above by a layer of permeability 10^{-17} m^2, then with a porosity of 0.1 and a layer inclination of 10^{o}, the along layer velocity scale U has value of order 10^{-8} m/s with fluid of viscosity 10^{-4} Pa s. If the overlying seal layer has thickness of order 1 m, then b has value of order 10^{-10} s^{-1} and so $\beta\tau \sim 1$, suggesting that the draining and the decay of the source occur over approximately comparable times. If the capillary entry pressure to the overlying layer is small, then the fraction of the flow retained in the layer is given by relation (32), within the simplified framework of this model, and this has value of about $F \sim 0.1$–0.2.

In the case of a rapidly decaying source or a current with slow drainage rate, $1 < \beta\tau$, then the drainage flux through the seal layer $F_D(x,t)$, per unit length along the current, which is supplied to points higher in the formation, is given by

$$F_D(x,t) = \beta h(0,0)\exp\left[-\frac{t}{\tau} + \frac{x}{U\tau}(1-\beta\tau)\right] \text{ for } 0 < x < Ut/\lambda$$
$$\text{if } t < t_c \text{ and for } \frac{U(t-t_c)}{1-\beta\tau} < x < Ut/\lambda \text{ if } t_c < t < t_d \tag{35}$$

While in the case of a slowly decaying source or current with high drainage rate, $\beta\tau > 1$, the drainage flux, again given by the same expression as in (35), is always located near to the source, with drainage in the region $0 < x < Ut/\lambda$ if $t < t_d$ and drainage in the region $0 < x < U(t - t_c)/(1 - \beta\tau)$ when $t_d < t < t_c$. Subsequently there is no draining.

With a capillary entry pressure corresponding to a depth of order 1 m, then with the above values for U, b and τ it follows that $t_c \sim t$ at which time the current has travelled a distance of order 100 m. The draining zone then evolves away from the source in the slow draining case, until time $t_d \sim (1.1$–$1.2)\, t_c$. Similarly in the fast draining case, the current will propagate about 100 m from the source, while the draining persists, with the illustrative parameters given in the example above. The plume will then cease to drain and will migrate along the original layer as a thin, elongate flow. For smaller capillary entry pressure, the flow may drain for times corresponding to several multiples of t, and hence the draining region may extend several hundred metres alongslope.

In a different situation of CO_2 sequestration, the injection period may only be of order 30 years. If the injectivity of the formation becomes impeded with time, the continuing injection flux may then decay with time, and the present model may give a guide to the flow. Initially, the flux per unit length injected into a long horizontal well may again be of order $10^{-5}\,m^2/s$, and if the formation has similar properties to the example above, this would correspond to the case $\beta\tau \sim 0.1$ which represents a rapidly decaying source compared to the drainage rate. Now the current would continue draining for a period $t_d \sim 5\tau$ which is about 150 years, in which time it would propagate about 50 m from the source. We note that in the case of a maintained steady flux, the drainage dynamics are somewhat different, as described by Woods and Farcas (2009).

From both these idealized examples, we see that with a decaying source, the injected fluid may rapidly spread alongslope and hence thin out, thereby limiting the fraction of the flow which drains into the overlying formation compared to the fraction which becomes capillary trapped in the original flowing layer.

6. Summary

Using a simplified approach, we have identified and modeled some of the controls on the migration of buoyant fluid through a layered permeable rock issuing from a waning source of buoyant fluid. We have focussed on the dynamics of the current in a single layer of the formation, examining the balance between leakage from the layer, and lateral spreading of the current along that layer.

In modelling the leakage, we have accounted for the capillary entry pressure into an overlying seal layer, and shown that this leads to a localized region of leakage which evolves in time. As the current wanes, the capillary entry pressure suppresses further leakage and the remainder of the current migrates through the original layer. Capillary trapping of the fluid in the original layer leads to a continual loss of fluid from the flow, and as the plume disperses, it is eventually trapped within this layer. The balance between the fraction of the flow which is trapped in the original layer, and the fraction which leaks into the overlying layer depends on the ratio of draining time through the overlying layer compared to the decay time of the source. With a rapidly decaying source, most of the fluid remains trapped in the original layer, whereas with a slowly decaying source, much of the fluid is able to leak into higher parts of the geological formation.

We have also shown that if the current reaches a fracture, then a significant part of this current may be diverted through the fracture and then migrate higher into the formation. The critical controlling parameter in this case is the ratio of the distance of the fracture from the source to the product of the Darcy flux and the decay time of the source. The further the fracture from the source the greater the fraction of the flow which is sequestered in the original layer in which the buoyant fluid is injected.

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